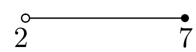
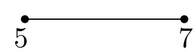
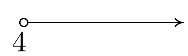
1.1.4 EXERCISES

To see all of the help resources associated with this section, click OSttS Chapter 1a.

1. Fill in the chart below. For help, click using interval notation.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \mid -1 \leq x < 5\}$		
	$[0, 3)$	
		
$\{x \mid -5 < x \leq 0\}$		
	$(-3, 3)$	
		
$\{x \mid x \leq 3\}$		
	$(-\infty, 9)$	
		
$\{x \mid x \geq -3\}$		

In Exercises 2 - 7, find the indicated intersection or union and simplify if possible. Express your answers in interval notation. For help, click using interval notation.

2. $(-1, 5] \cap [0, 8)$
3. $(-1, 1) \cup [0, 6]$
4. $(-\infty, 4] \cap (0, \infty)$
5. $(-\infty, 0) \cap [1, 5]$
6. $(-\infty, 0) \cup [1, 5]$
7. $(-\infty, 5] \cap [5, 8)$

In Exercises 8 - 19, write the set using interval notation. For help, click using interval notation.

8. $\{x \,|\, x \neq 5\}$

9. $\{x \,|\, x \neq -1\}$

10. $\{x \,|\, x \neq -3,\, 4\}$

11. $\{x \,|\, x \neq 0,\, 2\}$

12. $\{x \,|\, x \neq 2,\, -2\}$

13. $\{x \,|\, x \neq 0,\, \pm 4\}$

14. $\{x \,|\, x \leq -1 \text{ or } x \geq 1\}$

15. $\{x \,|\, x < 3 \text{ or } x \geq 2\}$

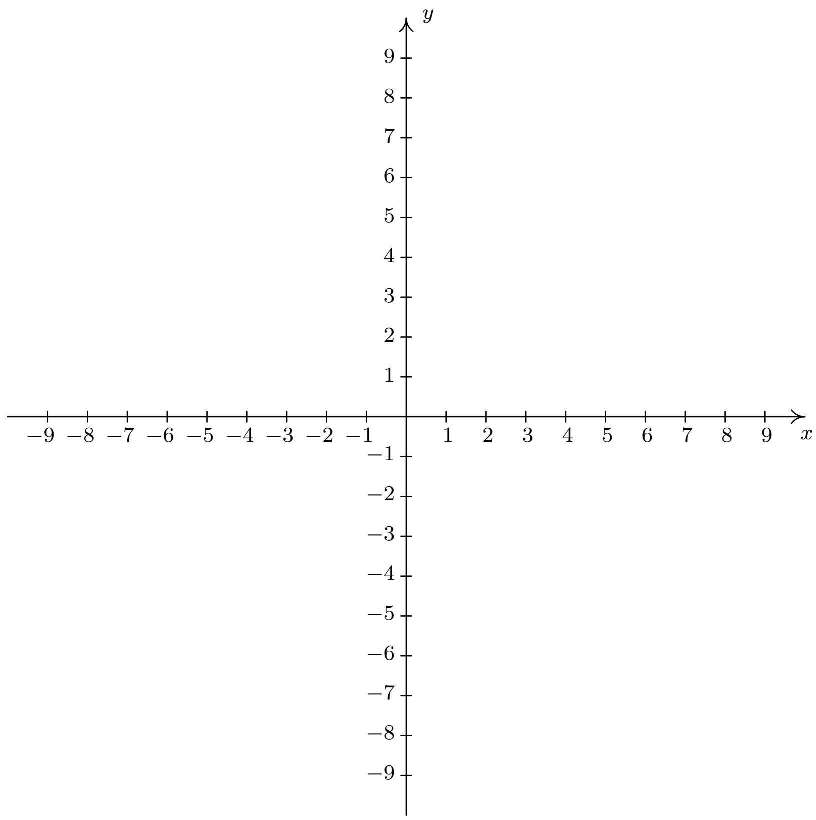
16. $\{x \,|\, x \leq -3 \text{ or } x > 0\}$

17. $\{x \,|\, x \leq 5 \text{ or } x = 6\}$

18. $\{x \,|\, x > 2 \text{ or } x = \pm 1\}$

19. $\{x \,|\, -3 < x < 3 \text{ or } x = 4\}$

20. Plot and label the points $A(-3,-7)$, $B(1.3,-2)$, $C(\pi,\sqrt{10})$, $D(0,8)$, $E(-5.5,0)$, $F(-8,4)$, $G(9.2,-7.8)$ and $H(7,5)$ in the Cartesian Coordinate Plane given below. For help, click plotting points in the plane.



For help with Exercise 21 below, click on one or more of the resources below:

- Identifying in which quadrant a point lies
- Finding points symmetric about the x-axis, y-axis, and origin

21. For each point given in Exercise 20 above
    - Identify the quadrant or axis in/on which the point lies.
    - Find the point symmetric to the given point about the x-axis.
    - Find the point symmetric to the given point about the y-axis.
    - Find the point symmetric to the given point about the origin.

In Exercises 22 - 29, find the distance d between the points and the midpoint M of the line segment which connects them.

For help with these exercises, click on one or more of the resources below:

- Using the distance formula
- Using the midpoint formula
- Link to prerequisite algebra material (For help simplifying radicals (square roots.))

22. $(1, 2)$, $(-3, 5)$
23. $(3, -10)$, $(-1, 2)$
24. $\left(\frac{1}{2}, 4\right)$, $\left(\frac{3}{2}, -1\right)$
25. $\left(-\frac{2}{3}, \frac{3}{2}\right)$, $\left(\frac{7}{3}, 2\right)$
26. $\left(\frac{24}{5}, \frac{6}{5}\right)$, $\left(-\frac{11}{5}, -\frac{19}{5}\right)$.
27. $\left(\sqrt{2}, \sqrt{3}\right)$, $\left(-\sqrt{8}, -\sqrt{12}\right)$
28. $\left(2\sqrt{45}, \sqrt{12}\right)$, $\left(\sqrt{20}, \sqrt{27}\right)$.
29. $(0, 0)$, (x, y)
30. Find all of the points of the form $(x, -1)$ which are 4 units from the point $(3, 2)$.
31. Find all of the points on the y-axis which are 5 units from the point $(-5, 3)$.
32. Find all of the points on the x-axis which are 2 units from the point $(-1, 1)$.
33. Find all of the points of the form $(x, -x)$ which are 1 unit from the origin.
34. Let's assume for a moment that we are standing at the origin and the positive y-axis points due North while the positive x-axis points due East. Our Sasquatch-o-meter tells us that Sasquatch is 3 miles West and 4 miles South of our current position. What are the coordinates of his position? How far away is he from us? If he runs 7 miles due East what would his new position be?

35. Verify the Distance Formula 1.1 for the cases when:
    (a) The points are arranged vertically. (Hint: Use $P(a, y_0)$ and $Q(a, y_1)$.)
    (b) The points are arranged horizontally. (Hint: Use $P(x_0, b)$ and $Q(x_1, b)$.)
    (c) The points are actually the same point. (You shouldn't need a hint for this one.)

36. Verify the Midpoint Formula by showing the distance between $P(x_1, y_1)$ and M and the distance between M and $Q(x_2, y_2)$ are both half of the distance between P and Q.

37. Show that the points A, B and C below are the vertices of a right triangle.
    (a) $A(-3, 2)$, $B(-6, 4)$, and $C(1, 8)$
    (b) $A(-3, 1)$, $B(4, 0)$ and $C(0, -3)$

38. Find a point $D(x, y)$ such that the points $A(-3, 1)$, $B(4, 0)$, $C(0, -3)$ and D are the corners of a square. Justify your answer.

39. Discuss with your classmates how many numbers are in the interval $(0, 1)$.

40. The world is not flat.[12] Thus the Cartesian Plane cannot possibly be the end of the story. Discuss with your classmates how you would extend Cartesian Coordinates to represent the three dimensional world. What would the Distance and Midpoint formulas look like, assuming those concepts make sense at all?

Checkpoint Quiz 1.1

1. Let $P(2, -3)$ and $Q(-5, -1)$
    (a) Plot P and Q.
    (b) Determine the quadrants in which P and Q lie.
    (c) Find the distance between P and Q.
    (d) Find the midpoint of the line segment which connects P and Q.
    (e) Find the point symmetric to P about the:
        i. x-axis
        ii. y-axis
        iii. origin

2. Find all points on the y-axis which are 3 units from $(-1, 5)$.

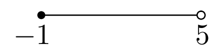
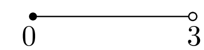
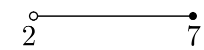
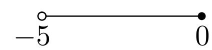
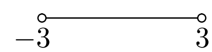
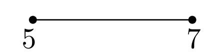
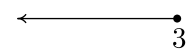
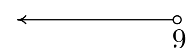
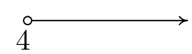
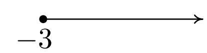
For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

[12] There are those who disagree with this statement. Look them up on the Internet some time when you're bored.

1.1.5 ANSWERS

1.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \mid -1 \leq x < 5\}$	$[-1, 5)$	
$\{x \mid 0 \leq x < 3\}$	$[0, 3)$	
$\{x \mid 2 < x \leq 7\}$	$(2, 7]$	
$\{x \mid -5 < x \leq 0\}$	$(-5, 0]$	
$\{x \mid -3 < x < 3\}$	$(-3, 3)$	
$\{x \mid 5 \leq x \leq 7\}$	$[5, 7]$	
$\{x \mid x \leq 3\}$	$(-\infty, 3]$	
$\{x \mid x < 9\}$	$(-\infty, 9)$	
$\{x \mid x > 4\}$	$(4, \infty)$	
$\{x \mid x \geq -3\}$	$[-3, \infty)$	

2. $(-1, 5] \cap [0, 8) = [0, 5]$

3. $(-1, 1) \cup [0, 6] = (-1, 6]$

4. $(-\infty, 4] \cap (0, \infty) = (0, 4]$

5. $(-\infty, 0) \cap [1, 5] = \emptyset$

6. $(-\infty, 0) \cup [1, 5] = (-\infty, 0) \cup [1, 5]$

7. $(-\infty, 5] \cap [5, 8) = \{5\}$

8. $(-\infty, 5) \cup (5, \infty)$

9. $(-\infty, -1) \cup (-1, \infty)$

10. $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$

11. $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

12. $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

13. $(-\infty, -4) \cup (-4, 0) \cup (0, 4) \cup (4, \infty)$

14. $(-\infty, -1] \cup [1, \infty)$

15. $(-\infty, \infty)$

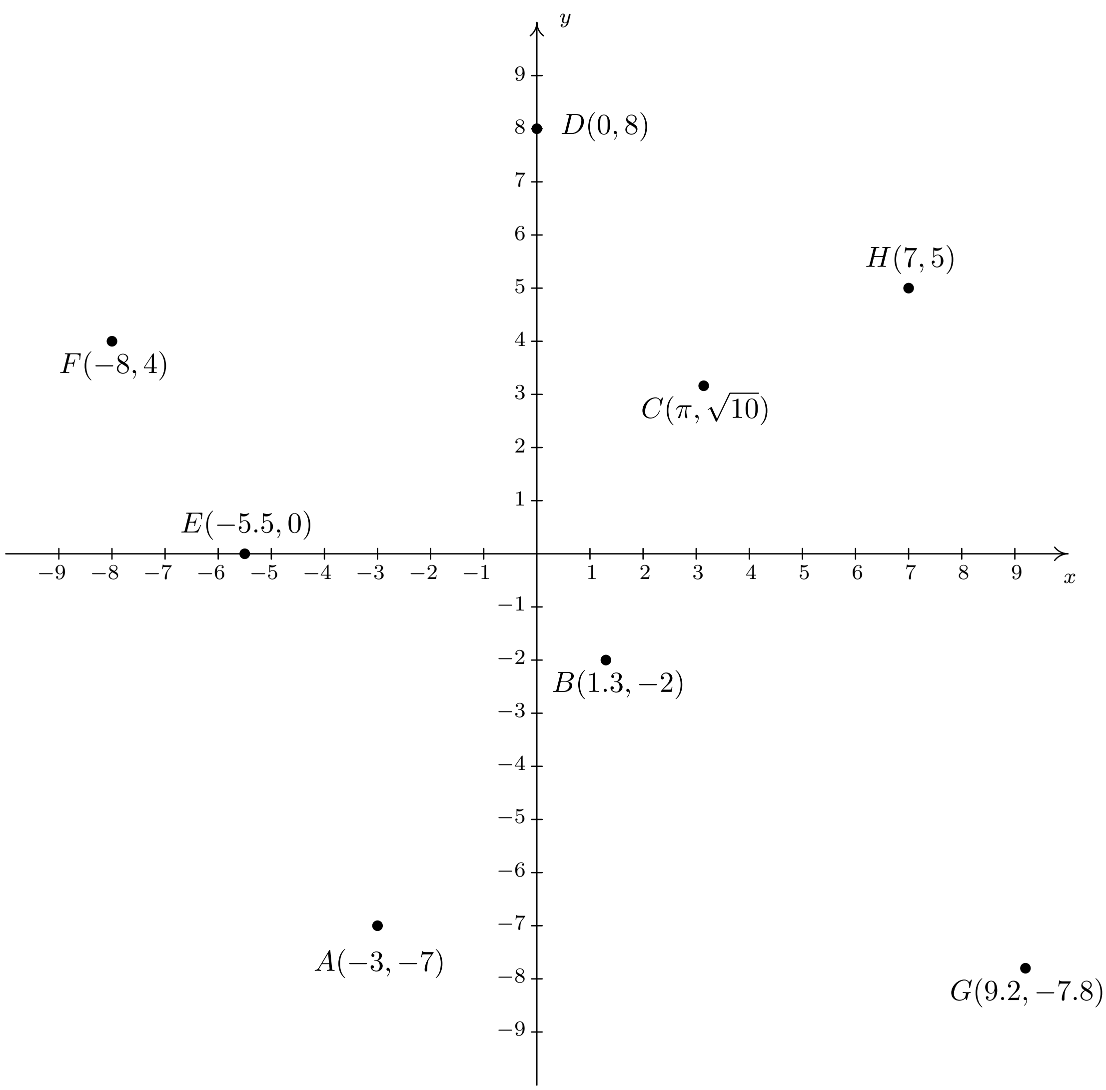
16. $(-\infty, -3] \cup (0, \infty)$

17. $(-\infty, 5] \cup \{6\}$

18. $\{-1\} \cup \{1\} \cup (2, \infty)$

19. $(-3, 3) \cup \{4\}$

20. The required points $A(-3, -7)$, $B(1.3, -2)$, $C(\pi, \sqrt{10})$, $D(0, 8)$, $E(-5.5, 0)$, $F(-8, 4)$, $G(9.2, -7.8)$, and $H(7, 5)$ are plotted in the Cartesian Coordinate Plane below.



21. (a) The point $A(-3,-7)$ is
    - in Quadrant III
    - symmetric about x-axis with $(-3,7)$
    - symmetric about y-axis with $(3,-7)$
    - symmetric about origin with $(3,7)$

    (b) The point $B(1.3,-2)$ is
    - in Quadrant IV
    - symmetric about x-axis with $(1.3,2)$
    - symmetric about y-axis with $(-1.3,-2)$
    - symmetric about origin with $(-1.3,2)$

    (c) The point $C(\pi,\sqrt{10})$ is
    - in Quadrant I
    - symmetric about x-axis with $(\pi,-\sqrt{10})$
    - symmetric about y-axis with $(-\pi,\sqrt{10})$
    - symmetric about origin with $(-\pi,-\sqrt{10})$

    (d) The point $D(0,8)$ is
    - on the positive y-axis
    - symmetric about x-axis with $(0,-8)$
    - symmetric about y-axis with $(0,8)$
    - symmetric about origin with $(0,-8)$

    (e) The point $E(-5.5,0)$ is
    - on the negative x-axis
    - symmetric about x-axis with $(-5.5,0)$
    - symmetric about y-axis with $(5.5,0)$
    - symmetric about origin with $(5.5,0)$

    (f) The point $F(-8,4)$ is
    - in Quadrant II
    - symmetric about x-axis with $(-8,-4)$
    - symmetric about y-axis with $(8,4)$
    - symmetric about origin with $(8,-4)$

    (g) The point $G(9.2,-7.8)$ is
    - in Quadrant IV
    - symmetric about x-axis with $(9.2,7.8)$
    - symmetric about y-axis with $(-9.2,-7.8)$
    - symmetric about origin with $(-9.2,7.8)$

    (h) The point $H(7,5)$ is
    - in Quadrant I
    - symmetric about x-axis with $(7,-5)$
    - symmetric about y-axis with $(-7,5)$
    - symmetric about origin with $(-7,-5)$

22. $d=5$, $M=\left(-1,\frac{7}{2}\right)$

23. $d=4\sqrt{10}$, $M=(1,-4)$

24. $d=\sqrt{26}$, $M=\left(1,\frac{3}{2}\right)$

25. $d=\frac{\sqrt{37}}{2}$, $M=\left(\frac{5}{6},\frac{7}{4}\right)$

26. $d=\sqrt{74}$, $M=\left(\frac{13}{10},-\frac{13}{10}\right)$

27. $d=3\sqrt{5}$, $M=\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{3}}{2}\right)$

28. $d=\sqrt{83}$, $M=\left(4\sqrt{5},\frac{5\sqrt{3}}{2}\right)$

29. $d=\sqrt{x^2+y^2}$, $M=\left(\frac{x}{2},\frac{y}{2}\right)$

30. $(3+\sqrt{7},-1)$, $(3-\sqrt{7},-1)$

31. $(0,3)$

32. $(-1+\sqrt{3},0)$, $(-1-\sqrt{3},0)$

33. $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$, $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

34. $(-3,-4)$, 5 miles, $(4,-4)$

37. (a) The distance from A to B is $|AB|=\sqrt{13}$, the distance from A to C is $|AC|=\sqrt{52}$, and the distance from B to C is $|BC|=\sqrt{65}$. Since $\left(\sqrt{13}\right)^2+\left(\sqrt{52}\right)^2=\left(\sqrt{65}\right)^2$, we are guaranteed by the converse of the Pythagorean Theorem that the triangle is a right triangle.

    (b) Show that $|AC|^2+|BC|^2=|AB|^2$